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G.K.BATCHELOR  
The theory of  
homogeneous  
turbulence

# THE THEORY OF HOMOGENEOUS TURBULENCE

BY

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## CONTENTS

PREFACE	page ix
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### I Introduction

1.1 The study of homogeneous turbulence	1
1.2 Mathematical formulation of the problem	3
1.3 Brief history of the subject	7

### II Mathematical Representation of the Field of Turbulence

2.1 Method of taking averages	14
2.2 The complete statistical specification of the field of turbulence	17
2.3 Mean values of velocity products	19
2.4 General properties of the velocity correlation and spectrum tensors	23
2.5 Fourier analysis of the velocity field	28

### III The Kinematics of Homogeneous Turbulence

3.1 The velocity correlation and spectrum tensors	34
3.2 The vorticity correlation and spectrum tensors	38
3.3 Symmetry conditions	40
3.4 Isotropic turbulence	45

### IV Some Linear Problems

4.1 Simple harmonic oscillator subject to a random force	55
4.2 Passage of a turbulent stream through wire gauze	58
4.3 Effect of sudden distortion of a turbulent stream	68

### V The General Dynamics of Decay

5.1 Methods of using the Navier-Stokes equation	76
5.2 The flow of energy	82
5.3 The permanence of big eddies	88
5.4 The final period of decay	92
5.5 Dynamical equations for isotropic turbulence	99

## VI The Universal Equilibrium Theory

<b>6.1</b>	The hypothesis of statistical equilibrium	<i>page</i> 103
<b>6.2</b>	Turbulent motion at large Reynolds number	106
<b>6.3</b>	The hypothesis of independence of Fourier components for distant wave-numbers	109
<b>6.4</b>	The universal equilibrium	114
<b>6.5</b>	The inertial subrange	121
<b>6.6</b>	The energy spectrum in the equilibrium range	125

## VII Decay of the Energy-containing Eddies

<b>7.1</b>	The decay of total energy	133
<b>7.2</b>	Evidence for the existence of a unique statistical state of the energy-containing eddies	139
<b>7.3</b>	The quasi-equilibrium hypothesis	148
<b>7.4</b>	The equilibrium at large wave-numbers for moderate Reynolds numbers	155
<b>7.5</b>	Heisenberg's form of the energy spectrum in the quasi- equilibrium range	161

VIII The Probability Distribution of  $\mathbf{u}(\mathbf{x})$ 

<b>8.1</b>	The experimental evidence	169
<b>8.2</b>	The hypothesis of a normal distribution of the velocity field associated with the energy-containing eddies	174
<b>8.3</b>	Determination of the pressure covariance	177
<b>8.4</b>	The small-scale properties of the motion	183

BIBLIOGRAPHY OF RESEARCH ON HOMOGENEOUS  
TURBULENCE

188

## INDEX

196